FSBA Dues

Buchholz Math Team Budget Proposal Version 2

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1 Problem

1.1 Background

Every school district in Florida must pay a certain amount of money to the Florida Department of Education. The FSBA Dues distribution has not been calibrated with enrollment size and other current variables for a significant amount of time. Since then, numbers have changed and there are marked anomalies in the current budget.

1.2 Guidelines

The Buchholz Math Team has been presented with data to configure a method of recalibrating the FSBA Dues to the current numbers and create a generalized formula that can be used as data changes over time. However, there are political factors to keep in mind in the calibration.

1. Dues reflect enrollment and district size
2. Small districts are not crowded out by large districts
3. Taxpayers are willing and able to satisfy the dues

1.3 The Data

The Buchholz Math Team was given a spreadsheet titled that included counts for the following:

- **2015-2016 FSBA Dues** - The uncalibrated dues paid this past year by the corresponding district
- **Enrollment** - The number of students enrolled in public schools in the corresponding district
- **FSBA Dues/Student**
- **Population** - The number of people that live in the corresponding district
- **Operating Budget** - Mathematically is "Total Funding" - "Lottery/Recognition". Represents the total wealth available in a certain district with regards to education.

Figure 1 shows the current dues sorted in ascending order. The points are not evenly distributed in any justified manner indicating a potential anomaly in distribution.
Figure 2 shows a scatter plot of the enrollment to dues from the uncalibrated data. The plot seems to be following a trend, possibly logarithmic, but there are many clear anomalies to a fitted curve. More about the natural logarithm can be found in Appendix A.

Figure 3 shows the current enrollment sorted in ascending order. This follows a very exponential type curve as there are few large districts and many smaller districts. There seems to be no significant clustering, further cluster analysis could be made using machine learning techniques that are often inconclusive and of high complexity. To linearize the curve in Figure 3a, the natural logarithm (logarithm base e) was applied. As seen in Figure 3b, the plot is close to linear, which also reduces the spread of the plot.

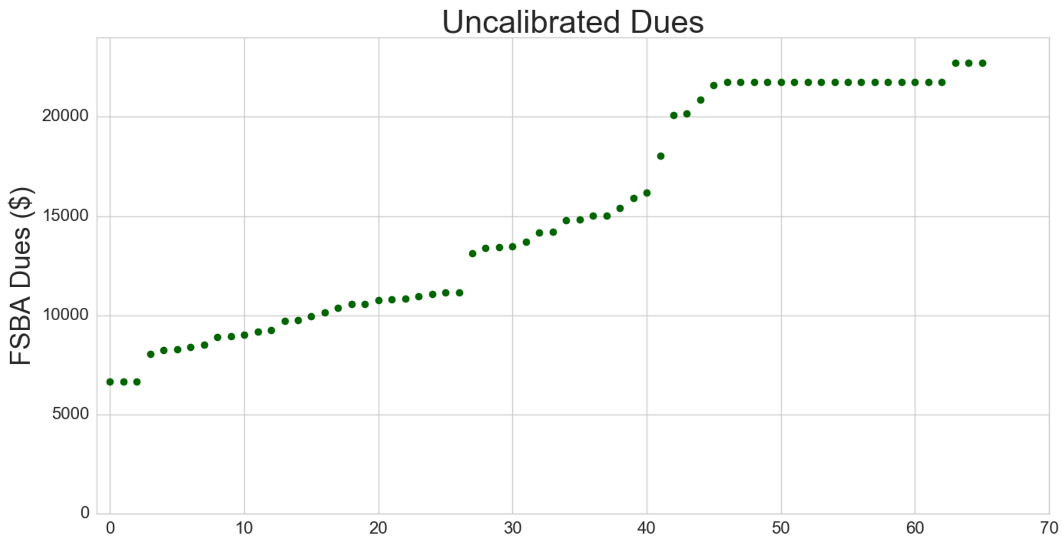


Figure 1: FSBA Uncalibrated Dues

A plot of the uncalibrated FSBA Dues from last year's numbers sorted in ascending order. Plots were created by Kim Nguyen using Python analytics with Matplotlib PyPlot and Seaborn.

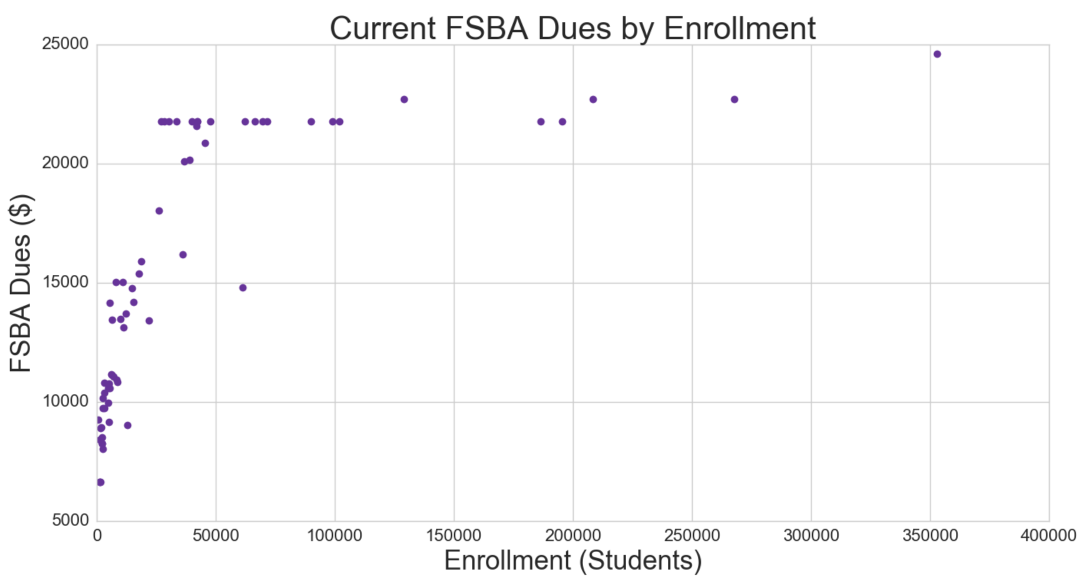
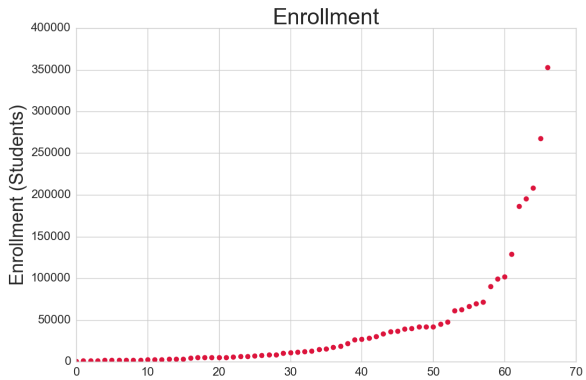
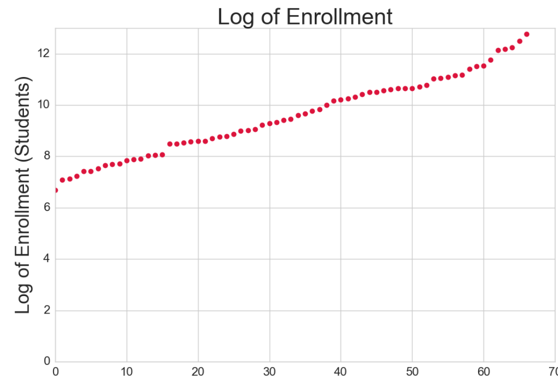


Figure 2: FSBA Uncalibrated Dues to Enrollment

A plot of the enrollment to FSBA Dues from last year's numbers. Plots were created by Kim Nguyen using Python analytics with Matplotlib PyPlot and Seaborn.



(a) Enrollment



(b) Log of Enrollment

Figure 3: Enrollment Distributions

Figure 3a is a plot of the enrollment sorted in ascending order. Figure 3b is the natural logarithm of the same data. Plots were created by Kim Nguyen using Python analytics with Matplotlib PyPlot and Seaborn.

2 Proposed Solution

The factors mentioned in Section 1.2 were in accounted for into a formula that is both justified and straightforward.

Our group took into account the ability for the district to pay dues by factoring in **Operating Budget** into our formula. This is to take into account the possibility that a county has a relatively high enrollment, but the taxpayers in that county may be financially unwilling to increase their share in taxes due to an increase in FSBA dues.

Define α as a district's *Average Ability to Pay*.

Mathematically, we defined this as:

$$\alpha = \frac{\text{Operating Budget}}{\text{Population}}$$

This value gives you the amount of tax dollars available for education per person in the entire district.

The function is as follows:

1. Calculate the Average Ability to Pay for each district.
2. Calculate the adjusted Average Ability to Pay p using the following for each district i:

$$p = \frac{\alpha_i}{\sum \alpha}$$

   (a) This gives the proportion of all Average Ability to Pay that a district represents.
   (b) All proportions sum to 1, and each proportion is a number between 0 and 1.

3. We create an intermediate index J with the following equation for each district i:

$$J = (\text{Enrollment})^2(p)$$

   (a) At heart, the FSBA budget is based off of enrollment, or size of the district, therefore α is simply used as a means of adjusting the enrollment to accommodate for the average wealth of the district.
   (b) The enrollment is weighted heavier than α with the use of the squaring factor.

4. Index J is put through the natural logarithm to obtain our final index I:

$$I = \ln(J)$$

   (a) See Figure 3a, enrollment in Florida is characterized by an exponential type curve, and is a strong fit to that curve.
   (b) The natural logarithm function is used to linearize this curve as seen in Figure 3b.

5. This final index determines the proportion of the desired \$1,021,000 each district pays.

3 Results

After implementing the above formula using Python 2.7, our group outputted `new_dues.xlsx`. This spreadsheet provides the new and old FSBA Dues, new and old Dues/Student, the α (Ability to Pay), and the Enrollment for each district.

The following plots visualize our outcome.

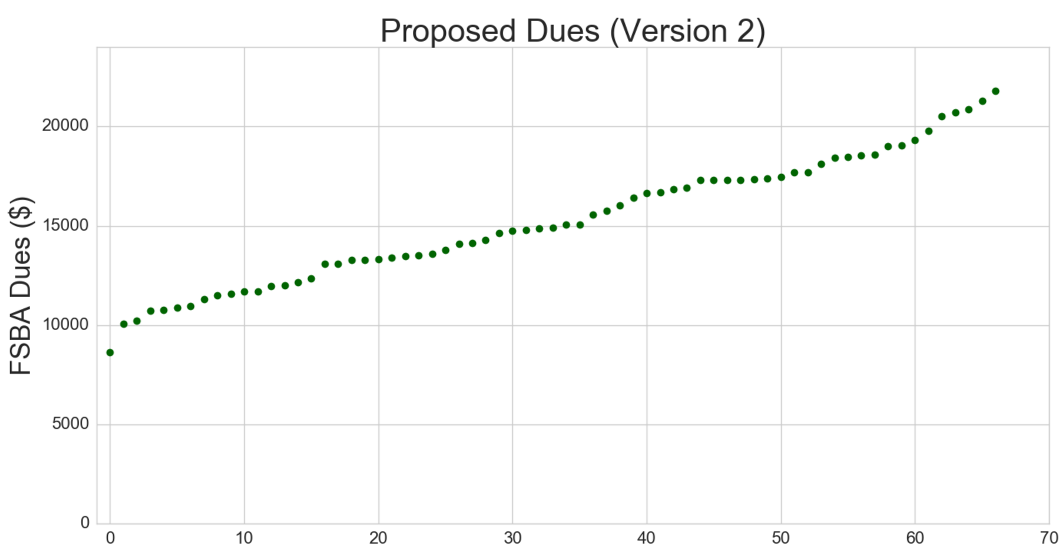


Figure 4: FSBA Calibrated Dues

A plot of the calibrated FSBA Dues obtained through our proposed formula sorted in ascending order. Plots were created by Kim Nguyen using Python analytics with Matplotlib PyPlot and Seaborn.

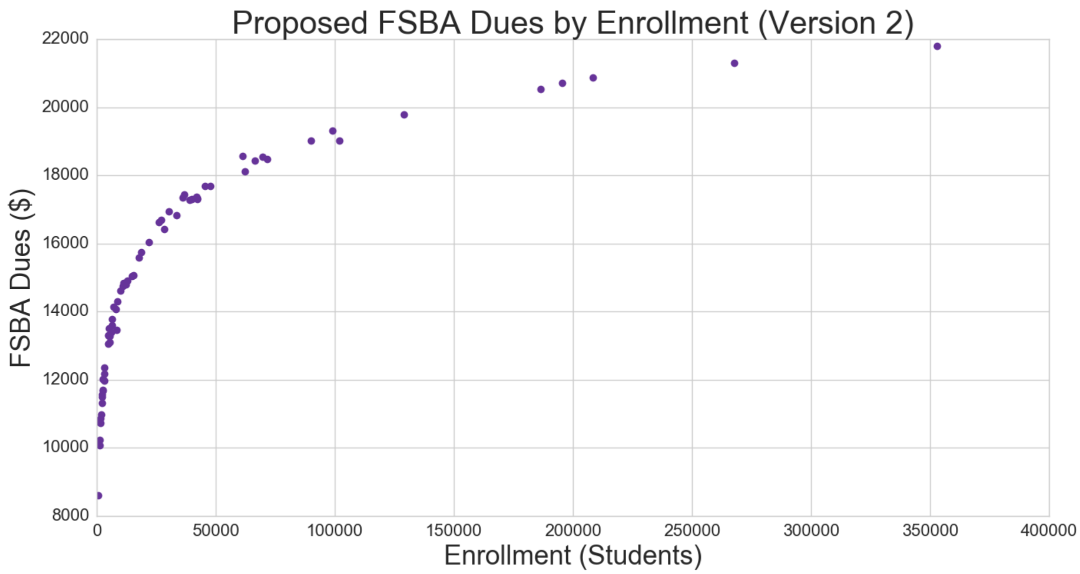


Figure 5: FSBA Calibrated Dues to Enrollment

A plot of the enrollment to FSBA Dues obtained through our proposed formula. Plots were created by Kim Nguyen using Python analytics with Matplotlib PyPlot and Seaborn.

Figures 4 and 5 can be compared to Figures 1 and 2 respectively as they share the same dimensions and scales. Overall, the plots generated from the proposed formula are more linear and fit to a trend much more strongly than do the plots from the uncalibrated data.

4 Conclusion

The FSBA Dues for the largest district, Miami-Dade, has gone from $24,621 to $21,783.53, which allows for a smaller say in the political scene by this large district.

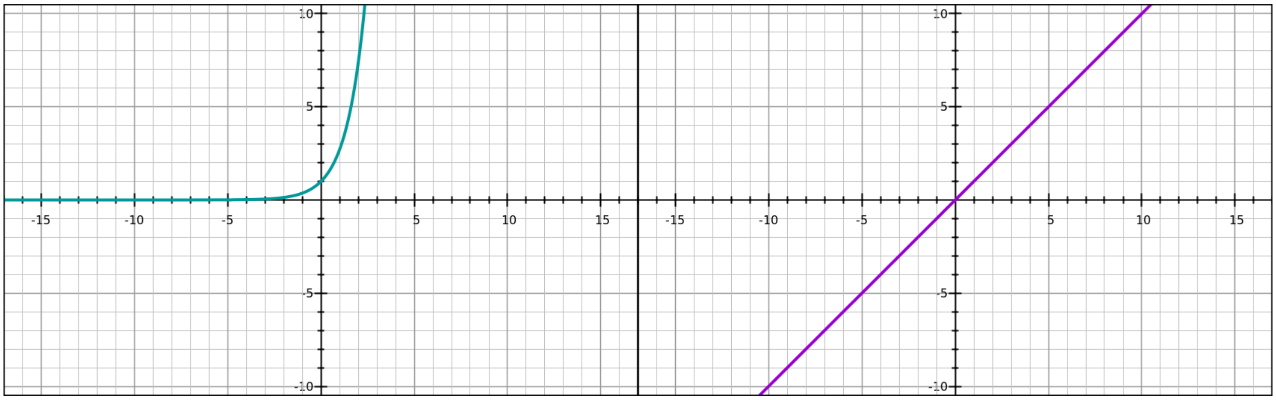
The FSBA Dues for the smallest district, Jefferson, has gone from $9,257 to $9,524.90, which is less drastic of a change, and allows for this district to remain in the political scene while not taxing the taxpayers excessive amounts of money for a small school district.

As a whole, the formula provides a simple method that incorporates a district's ability to pay the FSBA dues while understanding that a district's enrollment is a main factor.

A The Natural Logarithm

The log function, written as $y = \log_b x$, denotes the logarithmic function base b of x, which finds the exponent y such that $b^y = x$. For example, $\log_{10} 100$ would equal 2 because $10^2 = 100$. The ln function is the natural logarithm, which is the log function with the mathematical constant e (Approximately 2.72) as the base. For example $\ln e^4 = 4$ since the base, e is brought to the 4th power.

In our proposal, we use the natural logarithm to straighten out exponential curves. The following figure illustrates how the natural logarithm acts on exponential functions.



(a) Exponential Curve (b) Log of Exponential Curve

Figure 6: The Natural Logarithm Visualized

Figure 6a is the exponential curve e^x and Figure 6b is the function $f(x) = \ln(e^x)$, which is equivalent to $f(x) = x$. As you can see the curve goes from exponential to linear. Plots created with `graphsketch.com`.

For more on exponential functions and the natural logarithm, the following webpages are well written:

- [Exponential Functions]
- [The Natural Logarithm]